Abstract—Passivity analysis of any haptic system requires the knowledge of the environment impedance, i.e., parameters of the employed environment model. There have been a few models proposed to describe the viscoelastic behavior of soft tissues, including the popular Maxwell and Voigt models. This paper analyzes passivity of haptic systems interacting with virtual viscoelastic soft tissues. The Kelvin model is employed to represent better the behavior of the soft tissues. This passivity analysis reveals a new criterion for design and control of the haptic interface. Simulation results show that this new criterion increases the range of passive environment.

I. INTRODUCTION

PASSIVITY has been widely used for stability analysis of sampled data systems and haptic systems [1], [2]. The passivity analysis of a system requires the knowledge of environment parameters. Fardad and Bamieh [2] provided a criterion for the design of passive sampled-data systems in general and a haptic system in particular, whose results were similar to [1]. It has, however, been known that passivity is a more conservative criteria than the stability of a system [3]. Effects of various factors such as sensor quantization, velocity filtering, and human operator dynamics on the impedance of the haptic interface led to a modified criterion [4]. Minsky et al. [5] discusses issues such as effect of sampling time and human operator intervention in the force display and stability of haptic simulation. Diolaiti et al. [6] deals with effects of discretization, quantization, time delay, and coulomb friction on the stability of haptic rendering. Miller et al. [7] discusses the stability of haptic systems interacting with non-linear virtual environments. Effects of viscous damping and delay on the stability of haptic systems have been discussed by Gil et al. [8].

In medical simulations such as surgery simulation, haptic interfaces are used for interaction with the virtual soft tissues and organs. It is essential to have an accurate environment model to mimic the viscoelastic behavior of real soft tissues. Environment models such as the Maxwell and the Voigt models cannot accurately describe the viscoelastic behavior of soft tissues [9]-[12]. The Kelvin model, also referred to as the standard linear solid model [11] or Kelvin-Boltzmann body [12] in the literature, was proposed to overcome the disadvantages of the previous models.

This paper develops the criterion for passivity of haptic systems interacting with viscoelastic virtual soft-tissue environment. The developed criterion can serve as a guideline for the design of haptic interfaces employing the Kelvin model. It is shown that this new criterion increases range of the passive environment. This paper also analyzes the effect of discretization method on the passivity of the haptic systems.

II. VISCOELASTIC VIRTUAL ENVIRONMENT

Soft tissues are characterized by their viscoelastic behavior. The most commonly used Maxwell and Voigt models cannot account for the rate of dissipation of energy subject to cyclic loading [10]. The Maxwell model gives fluid-like behavior whereas the Voigt model gives more solid-like behavior. They are also inadequate for representing creep and relaxation behaviors [10]. Nonlinear models such as the Hunt-Crossley model are an extension of the linear Maxwell and Voigt models. They are expressed with variable stiffness and damping to feature large deformations or complex contacts between objects. However, these nonlinear models suffer from complexity and often lack physical or biological interpretation of the model parameters [12].
Viscoelastic behavior involves combination of instantaneous elastic response, delayed elastic response, and viscous flow \[ [11].\] The Kelvin model as shown in Fig. 1 combines aspects of the Maxwell and Voigt models to describe accurately the overall behavior of a system under a given set of loading conditions. It is the simplest model that can describe all the phenomena accurately \([10].\) The parallel spring in the Kelvin model \(\mu_0\) is responsible for the delayed elastic response and the series spring \(\mu_1\) describes the instantaneous elastic response of the environment. The damper \(\eta_1\) takes care of the viscous behavior. The modeled environment will instantaneously deform to some strain when a constant stress is applied exhibiting the elastic behavior. It will then continually deform and approach an asymptotic steady-state strain which is the viscous part. The creep and relaxation functions of the Kelvin model are shown in Fig. 2.

For the above model, let us break down the displacement \(u\) into \(u_1\) of dashpot and \(u'_1\) for series spring. The relations between force and displacement can therefore be expressed as given below \([10].\)

\[
\begin{align*}
F &= F_0 + F_1 \\
&= \mu_0 u + \eta_1 u_1
\end{align*}
\]

The realization of the displacement of the series spring \(u_1\) can be given by the following update law where \(T\) is the sampling time and \(u(k)\) is the position measurement from the device at step \(k\) \([9].\)

\[
\begin{align*}
u_1(k) &= \left( \frac{\eta_1}{T} u(k-1) + \frac{\eta_1}{T} u_1(k-1) \right) \left( \mu_1 + \frac{\eta_1}{T} \right)^{-1}
\end{align*}
\]

The calculation of the force \(F_1\) at the time instant \(k\) is thus known once the displacement \(u_1(k)\) is known by (1). Hence the total force can be calculated by (1) as given below.

\[
\begin{align*}
F(k) &= F_0(k) + F_1(k) \\
&= \mu_0 u(k-1) + \mu_1 u_1(k) \\
&= \mu_0 u(k-1) + \eta_1 \left( u(k-1) - u'_1(k) \right)
\end{align*}
\]

III. PASSIVITY ANALYSIS

A. Problem Formulation

A sampled-data haptic system is shown in Fig. 3, where the ZOH is a zero-order hold and \(T\) is the sampling time. Human operator manipulates the haptic interface with inertia \(m\) and damping \(b\) to interact with a virtual environment modeled by \(H(z)\). The dashed box is considered as a linear time-invariant system \(G(s)\), and hence, the state feedback connection of haptic interface is represented as follows \([2].\)

\[
\begin{align*}
\begin{bmatrix} z \\ y \end{bmatrix} &= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}
\end{align*}
\]

where \(G_{11}(s) = -1/(ms + b)\), \(G_{12}(s) = -1/(ms + b)\), \(G_{21}(s) = -1/(ms + b)\), \(G_{22}(s) = -1/(ms + b)\).

For this system, \([2]\) derived the necessary and sufficient condition for passivity as defined in (5).

\[
\begin{align*}
r_\theta \frac{K_\theta}{2b + r_\theta K_\theta} e^{-j\omega T} < 1
\end{align*}
\]

where \(r_\theta = -(1 - e^{-j\omega T}) \frac{T}{4\sin^2(\omega T/2)}\) and \(K_\theta = H(e^{j\omega T})\).

From (6) and \(b > 0\), the passivity condition (5) is formulated as (7).

\[
\begin{align*}
r_\theta &= -(1 - e^{-j\omega T}) \frac{T}{4\sin^2(\omega T/2)} \\
&= -(1 - e^{-j\omega T}) \frac{T}{2(1 - \cos \omega T)} \\
b &= \frac{T}{2(1 - \cos \omega T)} \Re \left\{ (1 - e^{-j\omega T}) H(e^{j\omega T}) \right\}
\end{align*}
\]

where the frequency \(\omega\) lies between 0 and Nyquist frequency \(\omega_N = \pi/T\). The condition (7) matches the passivity condition presented in \([1]\).

The relation between the applied force \(F\) and the total displacement \(u\) is described by (8). \(u_1\) and \(u'_1\) are the local displacements of the \(\eta_1\) and \(\mu_1\) respectively. The total displacement \(u\), which is also the displacement of \(\mu_0\), is the sum of the local displacements \(u_1\) and \(u'_1\).

\[
F + \frac{\eta_1}{\mu_1} (\acute{F}) = \mu_0 \left[ u + \frac{\eta_1}{\mu_1} \left( 1 + \frac{\mu_0}{\mu_1} \right) (\acute{u}) \right]
\]

Applying Laplace transform to the above equation, we have the following.
\[ F(s) + \frac{\eta_s}{\mu_s} (sF(s)) = \mu_s \left[ u(s) + \frac{\eta_s}{\mu_s} \left(1 + \frac{\mu_s}{\mu_0}\right)(su(s)) \right] \]  

(9)

The transfer function of the model \( H(s) \) is thus found out as follows.

\[ H(s) = \frac{F(s)}{u(s)} = \frac{\mu_0 + \eta_s \left(1 + \frac{\mu_s}{\mu_1}\right)}{1 + \frac{\eta_s}{\mu_1} s} \]  

(10)

The backward transformation is applied to convert (10) from continuous domain to discrete domain. The backward transform is given by

\[ s = \frac{1-z^{-1}}{T} \]  

(11)

Substituting (11) into (10) results in (12).

\[ H(z) = \frac{\mu_0 \mu_T + \eta \left(\mu_0 + \mu_s\right) \left(1-z^{-1}\right)}{\mu_1 T + \eta \left(1-z^{-1}\right)} \]

\[ = \frac{\mu_0 + \eta \mu_s \left(1-z^{-1}\right)}{\mu_1 T + \eta \left(1-z^{-1}\right)} \]

And, finally we get (13).

\[ H(z) = \mu_0 + \frac{1}{\mu_1 + \frac{Tz}{\eta \left(z-1\right)}} \]  

(13)

B. Passivity Criteria

Passivity criteria for the haptic system interacting with viscoelastic virtual environment is derived using the passivity condition (7) and the transfer function of the virtual environment modeled as Kelvin model. The pulse transfer function of the model, obtained from (13), is substituted in (7) to obtain (14).

\[ b > \frac{T}{2(1-\cos \omega T)} \Re \{ P \} \]  

(14)

where

\[ P = \frac{\mu_0 \left(1-e^{-j\omega T}\right) + \frac{1-e^{-j\omega T}}{\mu_1 \eta \left(e^{j\omega T} - 1\right)}}{\mu_1 + \frac{T e^{j\omega T}}{\eta \left(e^{j\omega T} - 1\right)}} \]  

(15)

\[ P \] can be simplified as follows.

\[ P = \mu_0 \left(1-e^{-j\omega T}\right) + \frac{e^{-j\omega T} \left(e^{j\omega T} - 1\right)}{\eta \left(e^{j\omega T} - 1\right) + \mu_T e^{j\omega T}} \]

\[ = \mu_0 \left(1-e^{-j\omega T}\right) + \frac{\mu_T \eta \left(e^{j\omega T} + e^{-j\omega T} - 2\right)}{\eta \left(e^{j\omega T} - 1\right) + \mu_T e^{j\omega T}} \]  

(16)

By using \( e^{j\omega T} = \cos(\omega T) + i \sin(\omega T) \),

\[ P = \mu_0 \left[(1-\cos \omega T) + i \sin \omega T\right] \]

\[ + \frac{2 \mu_0 \eta \left(\cos \omega T - 1\right)}{\eta \left(\cos \omega T - 1\right) + i \sin \omega T + \mu_T \left(\cos \omega T + i \sin \omega T\right)} \]

\[ = \mu_0 \left(1-\cos \omega T\right) \]

\[ + \frac{2 \mu_0 \eta \left(\cos \omega T - 1\right)}{\eta \left(\cos \omega T - 1\right) + \mu_T \cos \omega T} \]  

(17)

where,

\[ Q = \left[\eta \left(\cos \omega T - 1\right) + \mu_T \cos \omega T\right]^2 + \left(\eta \sin \omega T + \mu_T \sin \omega T\right)^2 \]  

(18)

This implies,

\[ \Re \{ P \} = \mu_0 \left(1-\cos \omega T\right) \]

\[ + \frac{2 \mu_0 \eta \left(\cos \omega T - 1\right)}{\eta \left(\cos \omega T - 1\right) + \mu_T \cos \omega T} \]  

(19)

Finally, (14) is written as (20) using (19).

\[ b > \frac{\mu_0 T}{2} \]

\[ - \frac{T \mu_0 \eta \left(\cos \omega T - 1\right) + \mu_T \cos \omega T}{\eta \left(\cos \omega T - 1\right) + \mu_T \cos \omega T} \]  

(20)

We then apply three extreme conditions for the frequency range. When \( \omega = 0 \), (20) becomes (21) as shown below.

\[ b > \frac{\mu_0 T}{2} \]

\[ - \frac{T \mu_0 \eta \left(\cos \omega T - 1\right)}{\left(\mu_0 T\right)^2} \]

\[ = \frac{\mu_0 T}{2} - \eta \]  

(21)

When \( \omega = \pi / 2 T \), (20) becomes (22),

\[ b > \frac{\mu_0 T}{2} + \frac{T \mu_0 \eta^2}{\left(\eta + \mu_T \right)^2 + \eta^2} \]  

(22)
When \( \omega = \pi / T \), (20) becomes (23),

\[
b > \frac{\mu_T T}{2} - \frac{\mu T \eta (-2\eta - \mu T)}{(-2\eta - \mu T)^2} = \frac{\mu_T T}{2} + \frac{\mu T \eta T}{2\eta + \mu T}.
\]

Equations (21), (22), and (23) are compared to give the maximum value that exists at \( \omega = \pi / T \) and is given in (23).

**Theorem 1:** A necessary and sufficient condition for passivity of haptic systems interacting with Kelvin models of virtual soft tissue, is given by (24). This condition is remarked as the criterion for viscoelastic passivity of haptic systems.

\[
b > \frac{\mu T}{2} + \frac{\mu T \eta T}{2\eta + \mu T}
\]

Dynamics of the human operator is not taken into account, and this dynamics makes the system more stable [4], [6]. It is also seen that humans can interact with passive objects in a stable fashion [13]. The human operator is generally passive but might be an active source of energy below 10 Hz [6]. It is, however, well known that the device inertia and friction together with the environment stiffness appear passive in the low frequency band. The interaction is hence stable at such low frequencies [6]. Coulomb friction and quantization are negligible since the haptic system components and the environment model are linear in nature [8]. Moreover, the coulomb friction can entirely dissipate the energy induced by the quantization at not-so-fast operational speeds of surgical simulation of soft tissues [6], [8], [14]. Time-delay may generate and inject energy but the coulomb friction of the haptic device also dissipates it at surgical speeds [6].

A condition for selection of sampling rate can also be derived based on (24). For a stiffer body, for which the delayed elastic response parameter, described by the parallel spring is high, the sampling rate has to be high. This result agrees with the common notion of the need of higher sampling rate when interacting with stiff bodies [1]. On the other hand, the sampling rate can be low for softer bodies with low parallel stiffness values.

**C. Effect of Discretization Method**

The popular bilinear transformation according to Tustin’s Method is used to convert (10) from continuous domain to discrete domain. The bilinear transform is given by

\[
s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}
\]

Therefore, the bilinear transform given in (25) is substituted in (10) to get (26).

\[
H(z) = \frac{\mu_0 + \frac{1}{\mu_T} T (z+1)}{\mu_0 + \frac{1}{\mu_T} T (z+1)}
\]

The above pulse transfer function of the environment model is substituted in (7) and simplified to get the necessary and sufficient condition for the passivity of sampled-data systems as given in (27) below.

\[
b > \frac{T}{2} (\mu_0 + \mu_T)
\]

Note that the minimum damping of the haptic device, which is required to maintain the passivity of the haptic system, is independent of the environment damping when the model is discretized using the bilinear transformation. This criterion is also more conservative when compared to (24) which is derived using backward transformation. Therefore, the passivity analysis highly depends on the choice of discretization method. It is to be noted, however, that the case of negative damping has not been considered in the analysis given above since virtual environment with negative parameters exhibit continuous growth in oscillation and is, thereby, unstable and impractical.

**IV. CASE STUDY**

Two sets of simulations for passivity analysis are conducted using MATLAB. The first set of simulations was carried out to see the effect of the environment parameters on the critically required damping of the haptic device to maintain the passivity of the system. Fig. 4 shows the critical damping for the haptic device required for passive interaction. The environment damping and the series stiffness varies from 5 kg/s to 50 kg/s, and from 10 N/m to 50 N/m, respectively. The environment parallel stiffness is taken to be 10 N/m. In Fig. 5, the environment series stiffness and the damping varies from 0 N/m to 50 N/m, and from 10 kg/s to 50 kg/s, respectively. The
Fig. 7. Range of environment parameters for passive system with variable $\eta$ and $\mu$ ($iP$ and $iVEP$ represent passivity and viscoelastic passivity when $b=i$ respectively).

Fig. 8. Range of environment parameters for passive system with variable $\eta$ and $\mu$ ($iVEP$ represents viscoelastic passivity when $b=i$).

The $K$ is the environment stiffness and the $B$ is the environment damping arranged in parallel.

The critical damping of the haptic device according to the viscoelastic passivity criteria does not vary as the environment damping changes as shown in Figs. 4, 5, and 6. This shows that the new criterion is independent of the environment damping. The critical damping is also seen to have very low magnitude and vary slightly with the variation of the environment series stiffness. This low magnitude of the critical damping allows the haptic device to be used for a larger range of environment.

The purpose of the second set of simulations is to find out the range of environment parameters for which the system will remain passive. The range of passive environment is found out based on the developed viscoelastic passivity criterion given in (24), and compared with the well known passivity criterion given in (28) based on Voigt model. Fig. 7 shows the range of environment according to the two criteria when the damping of the haptic device varies from 1 kg/s to 5 kg/s. The environment damping and the parallel stiffness varies from 0 to 1 kg/s and from 0 to 10 kN/m, respectively. The environment series stiffness is fixed at 50 N/m.

$$b > \frac{KT}{2} + B.$$  (28)
Let us consider the case when the damping of the haptic device is 3 kg/s. The area to the left of the line CA represents the range of passive environment according to the viscoelastic passivity criterion for the specified haptic device. Note that the area to the left of the line BA represents the range of passive environment according to the well-known passivity criterion given in (28). A closer look reveals that the new viscoelastic passivity criterion, that takes into account the viscoelastic behavior of soft-tissues, increases the range of the environment that the given haptic device can interact safely with. This enlargement is given by the area ABC.

Figs. 8 and 9 show the range of environment based on only viscoelastic passivity criterion with environment parallel stiffness fixed at 50 N/m and environment damping fixed at 1 kg/s, respectively. The figures show that the range of passive environment increases as the damping of the haptic device increases.

V. CONCLUSION

This paper develops a new passivity criterion based on the Kelvin model that better describes the viscoelastic behavior of soft tissues than the Maxwell and the Voigt models. It is shown that the developed viscoelastic passivity criterion increases the range of passive environment. This means that the haptic device with a given damping can safely interact with a larger range of viscoelastic virtual environment. The newly developed criterion is less conservative than the well-known previous passivity criterion based on Voigt model.

REFERENCES


